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| **TGC Fellow Unit Template**  |
| Prepared by: Erika Calvillo School/Location: Mozart Elementary / Chicago, IL,  |
| Subject: Mathematics Grade: 8th Interdisciplinary Unit Title: Functions: Linear Relationships and Exponential Growth and DecayTime Needed: 50 days  |
| **Unit Summary:** In this unit, that incorporate lessons from Thinking with Mathematical Models and Growing, Growing, Growing, students will understand that functions describe relationships and will be able to compare and construct a function. Students will recognize linearity in a table when constant differences between input values produce constant differences between output values. Students will establish a routine of exploring functional relationships algebraically, graphically, and numerically in tables and verbal descriptions. Students will learn that proportional relationships are part of a broader group of linear functions, with nonlinear functions for comparison. |
| **STAGE 1: Desired Results** |
| **ESTABLISHED GOALS:**Math CCSS:Thinking with Mathematical Models:* 8.EE.6 Use similar triangles to explain why the slop m is the same between any two distinct points on a non-vertical line in the coordinate plane, derive the equation y = mx + b for a line intercepting the vertical axis at b.
* 8.EE.7 Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressiosn using the distributive property and collecting like terms.
* 8.F.2 Compare properties of two functions each represened in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
* 8.F.3 Interpret the eqution y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.
* 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of tis graph or a table of values.
* 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a grpah. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
* 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative varialbes. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
* 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Growing, Growing, Growing:* 8.F.2 Compare properties of two functions each represened in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
* 8.F.3 Interpret the eqution y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.
* 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**GLOBAL COMPETENCY:**From Education for Global Competence: Preparing Our Youth to Engage the WorldStudents will: * Investigate the world beyond their immediate environment, framing significant problems and conducting well-crafted and age-appropriate research.
* Communicate ideas effectively with diverse audiences, bridging geographic, linguistic, ideological, and cultural barriers.
* Take action to improve conditions, viewing themselves as players in the world and participating reflectively.

From Oxfam’s “Maths and Global Citizenship”Students will be:* Provided with opportunities to illustrate mathematical concepts and processes by means of global issues and data
* Using and applying mathematics to real-world problems and data (for example, international development data)
* Provided with opportunities to consider the influence of different cultures on mathematics
* Developing critical thinking around use, presentation and manipulation of data

**TECHNOLOGY USED:** LCD projector, document camera, student laptops**RESOURCES:*** CMP2 textbooks Thinking with Mathematical Models and Growing, Growing, Growing
* Southern India Elephant Worksheet
* Real World Math: Engaging Students through Global Issues teacher guide and student workbook Lessons 1, 2, 9, 13
* Teachers Pay Teachers 21st Century Projects
* Facebook, Twitter, and the Arab Spring
* YouTube video Kony 2012
* Graph of the Week: Snapchat and Other Popular Social Media Platforms Among Teens in the U.S. (<http://www.turnersgraphoftheweek.com/>)
* Internet Safety and Media Advertising worksheet
* Bot or not? How fake media accounts affect democracy <http://www.pbs.org/newshour/extra/lessons_plans/lesson-plan-how-to-use-social-media-for-social-good/>
* Focus on factors that led to, sustained, and expedited the revolutions: the role of Arab youth; the use of social media; the role of the U.S. and other NATO countries; demographic factors such as the “youth#bulge”

<http://pulitzercenter.org/education/lesson-plan-writing-about-arab-spring>* TedTalk by Dalia Mogahed

<https://www.ted.com/talks/dalia_mogahed_the_attitudes_that_sparked_arab_spring>* Ted Talk Wael Ghonim: Inside the Egyptian Revolution <https://www.ted.com/playlists/12/freedom_rising>
* ePals Global Exchange – Students will connect with students in or around Egypt to ask questions and further learn about the Arab Spring
* Health vs. Wealth
* Graph of the Week: Life Expectancy and Health Care Spending (<http://www.turnersgraphoftheweek.com/>)
* Population Growth video <https://www.ted.com/talks/hans_rosling_on_global_population_growth>
* New Insights on Poverty video <https://www.ted.com/talks/hans_rosling_reveals_new_insights_on_poverty>
* Good News of the Decade video <https://www.ted.com/talks/hans_rosling_the_good_news_of_the_decade>
* Ted Talk Jacqueline Novogratz: An Escape from Poverty <https://www.ted.com/playlists/67/the_quest_to_end_poverty>
* Padlet for student collection of resources
* Social Media: Facebook and Instagram
* Exponential Modeling
* Who Wants to Be a Millionaire worksheet
* Paper Folding Activity
* “Radioactive” M&M worksheet
* Decaying Dice Game
* Population Growth worksheet
* African Black Rhino population worksheet

**MATERIALS:**Student CMP2 textbooks, paper, pencil, color pencils, calculators, graph paper | ***Transfer*** |
| *Students will be able to independently use their learning to:** To create and analyze graphs to make informed decisions.
* Interpret and persevere in solving complex problems using strategic thinking.
* Express appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and attending to precision when making mathematical statements.
* Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems.
* Based on an understanding of any problem, initiate a plan, execute it, and evaluate the reasonableness of the solution.
* Use appropriate tools to deepen understanding of mathematical concepts.
* Articulate math concepts effectively based on purpose, task, and audience using appropriate vocabulary.
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| ***Meaning*** |
| UNDERSTANDINGS*Students will understand how to:** Use real data on global issues with a focus on practical solutions.
* Investigate a range of global issues.
* Build on their mathematics knowledge wihle using critical thinking, collaboration with peers, and applying global perspectives.
* Build on their skills for future problem solving activites they will encounter as active, engaged citizens.

  | ESSENTIAL QUESTIONS:* How can understanding the nature of systems help us find solutions to large, complex problems?
* What is a mathematical model and how can it be used to solve problems?
* What variables create strong correlations with international data?
* What ways do we use math in everyday life?
* How can I best represent a pattern using mathematical principles?
* How could we use mathematics to create a better world?
* How do pictures, graphs, tables, and data “paint a thousand words?”
* What ways can mathematical problems conceal as much as they reveal?
* How do we identify patterns and use them to predict what will happen next?
* When and how has mathematics played pivotal roles throughout our world’s history?
* When solving multi-step problems using charts, tables, and graphs, how can you tell if the information you provide is sufficient?
* What is the difference between a linear equation and an exponential equation?
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| **Acquisition** |
| *Students will know how to:** Recognize linear and nonlinear patterns in tables and graphs and describe those patterns using words and equations.
* Write equations to express patterns appearing in tables, graphs, and “stories”.
* Solve linear equations.
* Model situations with inequalities.
* Write equations to describe inverse variation.
* Use linear and inverse equations to solve problems and make predictions.
* Recognzie situations in which one variable is an exponential function of another variable
* Recognize the connections between exponental patterns that appear in data tables, graphs, and problem conditions
* Understand and apply the rules for operating on numerical expressions with exponents
* Solve problems about exponential growth and decay from a variety of subject areas including science, social science, and business
* Compare exponential and linear relationships
 | *Students will be able to:** Make tables and graphs to represent data
* Describe relationships between variables
* Use data patterns to make predictions
* Compare and contrast linear and nonlinear relationships
* Fit a line to data tha show a linear trend
* Use mathematical models to answer questions about linear relationships
* Practice effective strategies for writing linear equations from verbal, numerical, or graphical information
* Develop skill in solving linear equations with approximation and exact reasoning methods
* Write inequatilites to represent “at most” situations
* Use equations to represent questions about problems and interpret the solutions in the context of the problem
* Explore situations that can be modleld by inverse variation relationships
* Investigate the nature of inverse variation in familiar contexts
* Compare inverse variations with linear relationships
* Gain an intutive understanding of basic exponential growth patterns
* Begin to recognize exponential patterns in tables, graphs, and equations
* Express a product of identical factors in both expoential form and standard form
* Write an equation for an exponential relationship
* Solve problems involving exponential growth/decay
* Compare different exponential growth/decay patterns
* Understand the role of the growth/decay factor in exponential relationships
* Make a table from the graph and equation of an exponential relationship
* Write equations for exponential relationships represented by tables and graphs
* Solve problems invovling exponential growth/decay
* Determine and interpret the y-intercept (initial value) for an exponential relationship
* Determine the growth/decay factor based on a verbal description of an exponential relationship
* Use sample population data to write an equation to model population growth
* Use knowledge of exponential relationships to make tables and graphs to write equations for exponential decay patterns
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| **Stage 2 - Evidence** |
| **Assessment** | **Evaluation Criteria (Learning target or Student Will Be Able To)** |
|  Assessments **FOR** Learning: | * KWL Chart: Before and after their initial data exploration, students will complete KWL charts on the topic.
* Observation: Students will be observed participating in discussion and problem-solving activities.
* Discussion: Students will articulate what they are doing as they work and will share ideas and solutions with their partner, group, and whole class. Students will discuss their findings, as well as the mathematics used to analyze data, and opinions based on evidence, in class or through social media.
* Exit Tickets: After each class session, students will write a reflection on the work that they did using 3, 2, 1 template (3 things that they learned, 2 ways they contributed to class, 1 question they still have or to consider how they would apply this concept or skill in a practical setting.
* Reflections: Students will reflect on their learning and how, if anything has impacted them.
* Graffiti Wall: Students will jot down facts, write personal opinions, and connect their learning to other areas of study based on weekly lessons.
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| Assessments **OF** Learning: | * Pre- and Post-Assessment: MARS Tasks
* Global Education Checklist pre- and post-unit
* Check-up Quizzes throughout the unit
* Padlet Collections: Students should have at least 4 categories with at least 5 links under each category for the topics discussed throughout the unit.
* Open-Response Questions: Teacher will assess students’ real-world understanding and how they analytical processes relate by using brief written response, a mathematical solution, a drawing, or a diagram, chart, or graph.
* Presentations: Each day, a different student will present complete solutions to the class periodically summarizing key concepts that were learned. Students will be assigned and the after the lesson students will provide a less than 5 minute presentation on the previous days’ learning, key points, and/or solutions to selected problems.
* Performance Assessment: Students will demonstrate what they have learned and how to solve problems through a collaborative effort in solving a complex problem together. Not only do they learn how to work in a team, but also how to brainstorm and utilize their separate gains of knowledge to benefit the whole.
* Culminating Project: Individual student math research project. Students will be provided with a list of initial ideas to select from to research, however, if students have other ideas, they will need teacher approval.
* Unit Test: Given at the end of the unit
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| **Stage 3 - Learning Plan** |
| **Pre-Assessment: MARS Task (1 day)**Students will complete the MARS Task Squares and Circles and complete KWL chart. Students will also complete a modified version of the Global Education Checklist. **Linear and Inverse Variation****Investigation 1: Exploring Data Patterns (3 days)**The three Problems in this first Investigation engage students in collecting and analyzing patterns in experimental data and in number sequences. They provide experiences with both linear and nonlinear phenomena and lay the groundwork for using mathematical functions as models of patterns and physical phenomena. Students encounter relationships, some linear and some nonlinear, that describe the relationships between variables such as bridge length and breaking weight. These models allow students to make better-supported conjectures about what will happen in real situations.**Quiz (1 day)****CCSS: 8.F.1, 8.F.2, 8.F.3**Students will be assessed on their ability to:* Identify the rules that take *x* as input and gives *y* as output is a function
* Compare two functions from different representations
* Identify linear functions that have a constant rate of change between any two points
* Use equations, graphs and tables to categorize functions as linear or non-linear

**Real World Math: Engaging Students through Global Issues Lesson 1: Waste and Recycling (1 day)**Students will be able to:* Identify number patterns
* Contstruct expressions that contain variables to represent real-world patterns
* Use tables and graphs to organize data
* Explore pattersn of waste disposal and recycling in the U.S.

**Investigation 2: Linear Models and Equations (5 days)**The five Problems in this Investigation review and extend student understanding and skill in formulating and applying appropriate linear function rules when given problem conditions and/or experimental data. The first Problem develops ideas for modeling linear data patterns, including use of residual analysis to measure the accuracy of a linear model. The next two Problems develop the concept of slope and then review techniques for writing linear expressions that match problem conditions, data tables, and graphs. The fourth and fifth Problems review and extend student understanding and skill in solving linear equations and inequalities by tabular, graphic, and symbolic methods.**Partner Quiz (1 day)****CCSS: 8.EE.7.b, 8.F.1, 8.F.3, 8.F.4**Students will be assessed on their ability to:* Create a model that estimates a pattern in a set of data
* Use a linear model to estimate
* Identify the slope of a linear model
* Write an equation using data from a given table
* Use equations to solve given situations
* Identify linear relationships and equations for lines that pass through two points, and one point with a given slope

**Real World Math: Engaging Students through Global Issues Lesson 2: Poverty and Microcredit (1 day)**Students will be able to:* Evaluate formulas for different values
* Solve multi-step equations
* Consider what it means to live in extreme poverty
* Investigate microcredit as a potential solution to extreme poverty

**Investigation 3: Inverse Variation (4 days)**As the title suggests, the Problems of this Investigation develop basic concepts of inverse variation including symbolic forms, rates of change, and graph patterns. Those ideas are developed and applied in three applied mathematical contexts—area of rectangles, distance/rate/time problems, and unit cost business problems. The final Problem of the Investigation extends the concept of modeling data to data with inverse variation characteristics.**Investigation 4: Variability and Association in Numerical Data (4 days)**The Problems of this Investigation review and enhance student understanding of and skill in data analysis by focusing on two key statistical concepts—correlation and standard deviation. Students explore positive and negative correlation shown in scatter plots. They use the correlation coefficient as a measure of the linear association between two variables. Finally, as a measure of spread in single variable distributions, they learn to calculate the standard deviation.**Investigation 5: Variability and Association in Categorical Data (3 days)**The Problems in this Investigation review and enhance student understanding of and skill in data analysis. The focus is on the use of two-way tables to test the strength of an association between categorical variables.**Quiz (1 day)****CCSS: 8.EE.7.b, 8.F.1, 8.F.3, 8.F.4**Students will be assessed on their ability to:* Write an equation for a line
* Match the graph with an equation
* Solve equations for x
* Make a table, graph, and write an equation for a real-life situation
* Identify slope of a line
* Explain what information the y-intercept of a line represents

**Real World Math: Engaging Students through Global Issues Lesson 9: Carbon Emissions (1 day)**Students will:* Graph an inequality on a number line
* Solve inequalities using multiplication and division
* Work with decimlas
* Recognize that certain activities release CO2 and other greenhouse gases
* Understand how CO2 is realted to global climate change
* Investigate actions to reduce our contributions to climate change

**Mid-Unit Research Project Introduction (ongoing)**Teacher will present students with a list of possible research topics based on concepts and skills around linear and exponential functions. Students can select from provided list of ideas of provide their own idea based on teacher approval. **Exponential Growth****Investigation 1: Exponential Growth (4 days)**Students will explore situations that involve repeated doubling, tripling, and quadrupling. Students will be introduced to one of the essential features of many exponential patterns: rapid growth. Students will make and study tables and graphs for exponential situations, describe the patterns they see, and write equations for them, looking for a general form of an exponential equation. Students will also compare and contrast linear and exponential patterns of growth. **Investigation 2: Examining Growth Patterns (3 days)**Students will focus on exponential relationships with y-intercepts greater than 1. The standard form of an exponential equation is y = a(bx). When x = 0, the equation becomes y = a since b0 = 1. Thus a, the coefficient of the exponential term, generally indicates the initial value of the exponentially growing quantity. This initial value is the y-value corresponding to x = 0, or the y-intercept. Each problem in the investigation presents information about an exponential pattern in a different form – in a verbal description, in an equation, and as a graph – helping students develop flexibility in moving among representations. **Quiz (1 day)****CCSS: 8.EE.1, 8.EE.4, 8.F2, 8.F.3, 8.F.5**Students will be assessed on their ability to:* Read and interpret a scenario by completing a table and its graph
* Write exponential and linear equations
* Use their equations to identify specific given values
* Interpret data patterns using a table, graph, or equation
* Write expressions in standard form

**Real World Math: Engaging Students through Global Issues Lesson 13: Systems and Global Education (2 days)**Students will be able to:* Graph linear functions
* Calculate slope
* Discover worldwide trends
* Use systems to solve problems

**Investigation 3: Growth Factors and Growth Rates (3 days)**Students will study non-whole number growth factors other than 1 and relate these growth factors to growth rates. As an example, students will consider money invested at 6% annual interest. In order to find the amount of money for a given year, multiply the amount from the previous year, multiply the amount from the previous year by 1.06. The growth factor in this case is 1.06, while the growth rate is 6% (or 0.06). Students will also explore how the growth rate and the initial value affect the growth pattern. **Quiz (1 day)****CCSS: 8.EE.1, 8.EE.4, 8.F2, 8.F.3, 8.F.5**Students will be assessed on their ability to:* Complete tables for an investment account and percent increase growth rate on a savings account
* Explain how the table was complete
* Determine growth rate
* Graph information from tables
* Describe the situations
* Make predictions based on tables or graphs
* Write equations based on a given situation

**Facebook, Twitter, and the Arab Spring (8 days)**Students will be able to:* Identify and use patterns to develop a function
* Compare rates of change
* Use data to make future predictions
* Analyze patterns of change
* Plot functions
* Compare mathematical rates of change with real world rates of change in international conflict

**Investigation 4: Exponential Decay (3 days)**Students will be introduced to exponential decay – patterns of change that exhibit successive, non-constant decreases rather than increases. These decreasing relationships are generated by repeated multiplication by factors between 0 and 1, called decay factors. Strategies for finding decay factors and initial population and for representing decay patterns are similar to those used for exponential growth patterns. **Who Wants to be a Millionare Activity (1 day)**Students will be able to: * Investigate the similarities and differences between and among constant, linear, and exponential functions
* Gain familiarity with the graphing calculator (how to use the **Stat** functions to put data in lists, find regression equations, and use **StatPlot, ZoomStat**, **Table**, and **Tblset functions**.)

**Health vs. Wealth (8 days)**Students will be able to:* Analyze the per capita GDP of 10 different nations
* Analyze data of Underweight Children and correlate it with 11 other measures to see what is related across 42 countries
* Analyze the cost and benefits of using short-term and long-term data while analyzing life expectancy in 13 different topics
* Analyze Child Mortality rates and per capita GDP measurements
* Research their own country and provide projections using 6 key variables of their choice **(see Lesson Plan below)**

**Exponential Modeling using Graphing Calculator (5 days)****Paper Folding Activity** Students will be able to: * Model exponential growth and exponential decay functions by folding paper
* Investigate how quickly an exponential function increases/decreases

**“Radioactive” M&M Activity**Students will be able to: * Collect experimental data from trials
* Use an exponential decay model will best represent the data, with a rate of decay close to 0.5

**Decaying Dice Game**Students will be able to: * Play a game to reinformce the concepts of exponential decay and probability of events

**Population Growth**Studetns will be able to:* Collect and analyze data from **U.S. Census Bureau International Database** to compare population growth in three countries, Ethiopia, U.S., China

**African Black Rhino Activity**Students will be able to: * Apply problem solving skills and what they have learned about exponential models to make predictions about future population figures for African rhinos

**Unit Test (1 day)****CCSS: 8.EE.1, 8.EE.4, 8.F2, 8.F.3, 8.F.5**Students will be assessed on their ability to:* Determine the equation of a given situation based on a table
* Determine the decay factor of a given relationship with an explanation of how the answer was determined
* Make predictions based on the given data
* Sketch and label graphs
* Determine fast/slow growing graphs
* Create tables, graphs, and equations for given situations
* Compare/contrast patterns of change
* Decide true/false statements based on given equations

**Research Project from Mid-Unit**Students will present their research project to a small group of students, focusing on why their decision to research that topic, findings, and conclusions.  |

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| **TGC FELLOWS UBD Lesson Template** |
| Lesson Title: Mathematical Models United Nations Country Research Subject: Mathematics Prepared by: Erika CalvilloMaterials Needed: Student laptops, document camera, LCD projector, speakers, pencils, graph paper, calculators, Country Research Guiding Questions handout, United Nations Protocol (student guidelines for presenting their data to class), Model United Nations Student Rubric, Glogster, Global Competency: Investigate the World, Communicate Ideas |
| **W**here is the lesson going?(Learning Target or SWBAT) | This lesson provides students with an introduction to a country research project of their selection using United Nations protocol when presenting their findings. Students will play the role of a social development analyst and an ambassador to a country of their choice, analyzing the countries data, based on 6 variables to project the world in 2020. Students will need to provide a clear picture of their countries many social issues and share mathematical models that project into 2020.  |
| **H**ook: | **T**ailored Differentiation: |
| After completing the Health vs. Wealth activities, students will make write down two things they wish they could change in the world based on the Global Goals and provide detailed reasons for their selection. Students will watch Hans Rosling video <https://www.ted.com/talks/hans_rosling_the_good_news_of_the_decade>. After watching the video, students will add details to their writing and/or make changes based on the video.  | Diverse Learners will only select 3 variables to research instead of 6.Bilingual students will have the option of completing their research in their native language. Teacher will demonstrate to bilingual students how to translate a websiteTeacher will circulate during research session and assist students as they work through the Country Guiding Questions handout.  |
| **E**quip: |
| Students will select a country to research and collect data based on 6 different variables to make projections for the world in 2020 or any other variables based on the Global Goals. Using Country Research Guiding Questions handout students will write down the data they have researched based on the selected 6 variables.  |
| **Rethink and revise:** |
| Teacher will circulate and assist students as needed. Teacher will share any valuable insights from specific students with the class. Students will begin to develop a visual that provides important information about their country and mathematical evidence, such as a scatterplot and best-fit function of what students believe is their most important finding.  |
| **Evaluate:**  |
| Using provided rubric, students will be assessed on the following: Presentation:Presenters & Audience - Half of the class will be presenting their countries, while the other half will serve as an audience. The roles will reverse the next day. Audience Responsibilities -- the Audience will take notes on the “Question Asking Template" for each of the countries and will be responsible for asking one question for either discussion or to a specific country. Each country will prepare a thirty-second send-off with an action step they can make to work toward the goal of ending hunger.Research: Students used class time wisely to research their country and have gained knowledge that includes correctly completed mathematical projections of the data, analyzes the data to make authentic, meaningful conclusions, and includes visual aids. |
| Resources: * Google Translate for bilingual students (<https://translate.google.com/>)
* Suggested, but not limited to, country research websites: <http://www.gapminder.org/>, <https://data.worldbank.org/>, <http://www.nationmaster.com/>, <http://www.fao.org/home/en/>
* Global Goals <http://www.globalgoals.org/>
* Health vs Wealth Teachers Pay Teachers 21st Century Projects
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| **O**rganization:  |
| Students will submit by email, Google docs, Glogster, or paper format the following artifacts:* a fact sheet for their country
* name plate of country
* visual aid that includes important country information, map, flag, with mathematical evidence

During presentation student can speak knowledgeably and accurately from their perspective, understand the data and the correlations to inform their point-of-view. Students will have access to calculators, laptops, or any other materials during class time to complete their research. Teacher will assist struggling math students. |